A Modified Method for Recovering Bathymetry from Altimeter Data

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Abstract. At present, there exist two types of methods to recover the bathymetry from altimeter data, i.e. the deterministic method and the stochastic one. This paper first reviews the general principles of the two aforementioned methods in order to form the basis for the development of the new approach. Then, based on the theory of least-squares collocation, a modified statistical model for recovering bathymetry from altimeter data is proposed. The new model is used to compute the ocean depth in the South China Sea from altimetry-derived gravity anomalies. Finally the predicted depths are compared to the shipborne ones. The results show that the achievable agreement is very good. Taking into account the existence of errors in the shipborne depths, it can be believed that the relative error of altimetry-derived depths reaches the level of about 7%.

Keywords. Altimetry, bathymetry, least-squares collocation

1 Introduction

With the advent of satellite altimetry it has been proved that this new technology had an impact not only to geodesy, but to other scientific fields as well, such as oceanography and geophysics. Satellite altimetry technique provides direct measurements of sea surface heights with respect to the reference ellipsoid, the geometrical reference surface of the Earth, and makes it possible to determine the gravity field of the oceans on a global scale. This set of data can be used to predict deep-seafloor bathymetric features. It is known that one of the major achievements in the use of satellite altimetry in geodesy has been the determination of the marine geoid with high accuracy and high resolution. After corrections for the sea surface topography and other disturbing factors, satellite altimetry can provide an estimation of the marine geoid with a level of precision of about 5~10 cm (Hwang et al., 1998). Such a precise geoid offers a good opportunity to determine the marine gravity field over the world's oceans. Recent articles include those by Sandwell and Smith (1997), Knudsen and Andersen (1997), Andersen and Knudsen (1998), and Hwang, Kao, and Parsons (1998).

Since geoid anomalies decay slowly with distance r from a mass anomaly (proportional to r^{-1}), the shape of the geoid reflects the distribution of a relatively deep-seated mass within the Earth. The gravity anomaly decreases more rapidly (proportional to r^{-2}) and hence is more sensitive to shallow mass anomalies. Nevertheless, a major contribution to the marine geoid comes from topographic anomalies in the very shallow rock-water interface at the base of the ocean, since this surface represents a large density contrast. Consequently, there is a strong correlation between the shape of the geoid and ocean bottom topography (bathymetry). Although this fact was first declared in the 19th century by Siemens (1876), who suggested that ocean depths could be inferred from sea surface gravimetry, it was more than a century later when Dixon et al. (1983) demonstrated that the technique was feasible. Since then, modeling bathymetry with satellite altimetry has been carried out by many researchers, e.g., Smith and Sandwell (1994), Ramillien and Cazenave (1997), Arabelos (1997), Hwang (1999), and Vergos and Sideris (2002). Calmant and Baudry（1996） give a comprehensive review of the techniques and data used in bathymetric modeling.

At present, there exist two types of methods to recover the bathymetry from altimeter data, i.e. the deterministic and the stochastic ones. The former is based on Parker's formula and the three plate models of Watts (1978), while the latter on the least-squares collocation derived from the stochastic point of view. The bathymetric model from altimeter data will not be sufficiently accurate for such purposes as maritime navigation and hazard

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prevention, but can be very useful in oceanography. In this paper, the principles of the two types of recovering method mentioned above are first introduced. Then, based on the theory of least-squares collocation, a modified stochastic model for recovering bathymetry from altimeter data is proposed. The new model is used to compute the ocean depth in the South China Sea from altimetry-derived gravity anomalies. Finally the predicted depths are compared to the shipborne ones. The results show that the achievable agreement is very good.

2 Description of the Depth Estimation Model

2.1 The Deterministic Method

Seafloor is the shallowest density interface in the oceanic domain. Depth variations of the seafloor can be considered as height variations of mass elements the density $\Delta\rho$ of which is given by the density contrast between rock and sea water. These seafloor depth variations generate variations to the local gravity field. According to Calmant and Baudry (1996), the disturbing potential $T(r)$ due to a given mass element of volume V is:

$$T(r) = G\Delta\rho\int_V \frac{dv}{|r-r'|} \tag{1}$$

where G is the gravitational constant, r is the coordinate vector of location at which the disturbing potential is computed and r' the coordinate vector of the center of the mass element. The Brun's formula relates the geoid height N to the disturbing potential T. The discretized form of the Brun's formula corresponding to Equation (1) is:

$$N(r) = \frac{G\Delta\rho}{\gamma}\sum_{r'}\Delta\Omega(r')\int_{Z_b}^{Z_t}\frac{dZ}{|r-r'|} \tag{2}$$

where γ is the normal gravity acceleration at the surface of the Earth, $\Delta\Omega(r')$ is the element of the integrated surface, z_b and z_t are the depths of the bottom and the top of the mass element centered on r', respectively, and the Z axis is vertical. Equation (2) is in a convolution form, thus, it can be evaluated in the Fourier domain. The Fourier Transform of Equation (2) is, in plane approximation (Parker，1972):

$$F[N(r)] = \frac{2\pi G\Delta\rho}{\gamma}\exp(-|k|z_0)\sum_{n=1}^{\infty}\frac{|k|^{n-2}}{n!}F[\mathrm{Z}^{\mathrm{n}}(\mathrm{r}')] \tag{3}$$

where r is now the vector formed by coordinates in the $x-y$ plane, k is the modulus of the wave number vector, z_0 is the reference depth of the density interface, and $Z(r')$ stands for the depth variations of the interface relative to z_0. Details concerning the derivation of the Fourier expansions can be found in Parker (1972). A linear form of Equation (3) is obtained by limiting the series to the first term:

$$F[N(r)] = \frac{2\pi G\Delta\rho}{\gamma}|k|^{-1}\exp(-|k|z_0)F[\mathrm{Z}(\mathrm{r}')] \tag{4}$$

The above linear approximation allows an easy recovery of the topography of an uncompensated interface $b(r)$ from a given geoid anomaly $N(r')$:

$$b(r) = F^{-1}\{[Z_N(k)]^{-1}F[\mathrm{N}(\mathrm{r}')]\} \tag{5}$$

with $$Z_N(k) = \frac{2\pi G\Delta\rho}{\gamma}|k|^{-1}\exp(-|k|z_0) \tag{6}$$

Similarly, when gravity anomalies $\Delta g(r')$ are available, $b(r)$ is recovered as:

$$b(r) = F^{-1}\{[Z_g(k)]^{-1}F[\Delta\mathrm{g}(\mathrm{r}')]\} \tag{7}$$

with $$Z_g(k) = 2\pi G\Delta\rho\exp(-|k|z_0) \tag{8}$$

where $Z_N(k)$ and $Z_g(k)$ are called the admittance functions. The above two sets of the equations including Equation (5) to (8) are considered as the basic formulas of the deterministic method to model bathymetry from altimetry-derived geoid height and gravity anomalies, respectively. According to Equation (7), the depth estimation model can be written in the following linear form in the frequency domain:

$$B(u,v) = K(u,v)\Delta G(u,v) \tag{9}$$

where B and ΔG are the Fourier transforms of depth and gravity anomaly, respectively, K is the transfer function, and u , v are the two spatial frequencies. In practical application, K can be set to (Hwang, 1999)

$$K(u,v) = \frac{1}{2\pi G\Delta\rho} e^{2\pi d\sqrt{u^2+v^2}} \tag{10}$$

where $\Delta\rho$ is the density contrast between Earth's upper lithosphere and sea water, and d is the mean depth of the interface between the two layers. The problem of predicting depths from gravity data is nothing more than the well-known downward continuation problem. This is clear from Eq.(10) where the gravity to bathymetry transfer function $K(u,v)$ is the downward continuation kernel function ($e^{2\pi d\sqrt{u^2+v^2}}$) multiplied by a constant number, i.e., $\frac{1}{2\pi G\Delta\rho}$. In the mathematical literature, this operation is regarded as an ill-posed problem, whose solution will be non-unique and unstable, thus, it requires some regularization treatment, see Tihknov and Arsenin (1977). Because of these problems, Smith and Sandwell (1994) simply removed components with wavelengths shorter than 15km in their final bathymetric model, because the gravity to topography transfer function becomes singular at wavelengths shorter than about 15km. Also, the spatial resolution of altimetry SSHs is about 6km at the best case (GEOSAT/GM), thus, based on Nyquist theorem, one can resolve frequencies of wavelengths longer than about 12km. Hwang (1999) provided a detailed procedure for computing a predicted bathymetric model.

2.2 The Stochastic Method

Based on Parker's formula (1972) and the three plate models of Watts (1978), the bathymetric model of the deterministic method can be constructed between the residual fields of depth and gravity anomaly. But, given the complex composition of the seafloor, hardly any of the three plate models of Watts (1978) will match reality, and even if they do match, parameters such as plate thickness and flexural rigidity are difficult to estimate correctly. Due to these problem, it is useful to model bathymetry using a stochastic approach. In 1994, Tscherning et al. experimented first with a least-squares collocation method in which gravity anomalies and a-priori given depths were used to produce new depths in an area where the depth data may be considered erroneous. A lot of similar works have been carried out in the following years for the same purpose. Arabelos (1997) and Vergos and Sideris (2002) made a study on the possibility to estimate ocean bottom topography from marine gravity and satellite altimeter data using collocation. Hwang (1999) used the following equation to construct his bathymetric model for the South China Sea (SCS):

$$h = \mu_h + C_{hg} C_{gg}^{-1} (\Delta g - \mu_{\Delta g}) \tag{11}$$

where h is depth, Δg is gravity anomaly, μ_h and $\mu_{\Delta g}$ are the expected values of depth and gravity anomaly, respectively, C_{hg} is the depth-gravity anomaly cross-covariance matrix, and C_{gg} is the gravity anomaly auto-covariance matrix. In practical applications, the expected values of depth and gravity anomaly can be represented by the regional trends. Let the residual depth and gravity anomaly be:

$$h_{res} = h - \mu_h \tag{12}$$

$$\Delta g_{res} = \Delta g - \mu_{\Delta g} \tag{13}$$

Then, we have from Equation (11):

$$h_{res} = C_{hg} C_{gg}^{-1} \Delta g_{res} \tag{14}$$

which is equivalent to the formula of least-squares collocation (Moritz,1980). The covariance matrices C_{hg} and C_{gg} can be constructed from covariance functions. However, it is rather difficult to model the cross-covariance function needed for C_{hg} due to the fact that h and Δg come from two different types of parameter field. Because of this reason, Hwang (1999) proposed to use the concept of frequency domain collocation to speed up the computations, in which the covariance functions were replaced by the power spectral density functions. Following this idea, he computed a corrected bathymetric model for SCS, which had a better agreement with the shipborne depths as compared to the predicted model.

2.3 The New Modified Model

According to the theory of least-squares collocation

(Moritz, 1980), Equation (14) originates from the following model:

$$\Delta g = (1,0)\begin{pmatrix} \mathrm{s} \\ \mathrm{h} \end{pmatrix} + n \tag{15}$$

where the observation Δg is decomposed into a "signal part" s and a "noise part" n. Obviously, Equation (15) does not give a deterministic functional relation between the gravity anomaly Δg and the depth h. In Equation (14), the link relating Δg and h is their cross-covariance function. In regional gravity field approximation, height information is usually used to improve the accuracy of predicted free-air gravity anomaly in a remove-compute-restore concept. The prediction model can be written as:

$$\Delta \mathrm{g} = a + bh + s + n \tag{16}$$

where a and b are the unknown parameters. A lot of researches (Moritz, 1980) have shown that the above prediction model could always give better anomaly estimates than the simple model (see Equation (15)). This fact gives the motive to modify the depth estimate model in a similar way. Suppose that there is a linear correlation between the gravity anomaly Δg and the depth h. Similar to Equation (16), the depth prediction model can be rewritten as:

$$h = a + b\Delta g + s + n \tag{17}$$

where a and b are still the unknown parameters, while s represents the "signal part" of observation h, and n represents the "noise part" of observation h. The observation equations can be expressed in matrix notation as follows:

$$L = AX + S + V \tag{18}$$

where L represents the depth observation vector, X is the unknown parameter vector, which is comprised of a and b, A is a given matrix expressing the effect of the parameters X on the observations L, which is comprised of 1 and Δg, S is the signal vector, and V is the correction vector of L. Equation (18) is the well-known model of least-squares collocation with parameters. The general minimum principle can be used to derive optimal estimates for X and S. The least square solutions of Equation (18) are (Moritz, 1980):

$$X = (A^T \overline{C}^{-1} A)^{-1} A^T \overline{C}^{-1} L \tag{19}$$

$$S = C_{\mathrm{ss}} \overline{C}^{-1} (L - AX) \tag{20}$$

$$\mathrm{V} = \mathrm{C}_{\mathrm{nn}} \overline{C}^{-1} (L - AX) \tag{21}$$

$$\mathrm{H} = L - V \tag{22}$$

where $\overline{C} = C_{ss} + C_{nn}$, which is called the total covariance matrix of L, C_{ss} and C_{nn} are the covariance matrices of the signal s and the noise n, respectively, and H is the adjusted depth vector. From Equation (19) to (22), we can obtain an improved prediction depth model using an iterative procedure, based on a given initial depth model. For the first iteration, the covariance matrix of the signal s should be replaced by that of the initial depth model h owing to the fact that the signal s is unknown in the first step. When the solution converges, i.e. the differences or corrections to the adjusted vector (H) are smaller than a given threshold, the iterative procedure is finished.

As mentioned above, the difference between Equation (15) and (17) is that the statistical relation between the gravity anomaly Δg and the depth h in the former model has been changed to a deterministic linear relation in the latter. The advantage of this modification is that the new model does not require the determination of the cross-covariance function needed for C_{hg}. The gain of the new model in computational efficiency is illustrated in the case study that follows.

3 A Case Study

To evaluate the new method proposed above, some practical computations and a comparison to depth sounding have been carried out. In the present case study, the computation area is defined as: ($2° \leqslant \varphi \leqslant 25°$; $105° \leqslant \lambda \leqslant 125°$). The data file of altimeter-derived gravity anomaly from Huang et al. (2001) has been used as input information. The gravity data are given on grid with 2′ spatial resolution and were derived from the altimeter data of Seasat, Geosat, ERS-1 and TOPEX/POSEIDON. The newly developed 5′ ×5′ global digital topographic model JGP95E (Lemoine, et, al., 1998) has been used as the initial depth model of iterative computation. The following formulas have been applied to compute the covariance matrix:

$$C(d) = \frac{1}{2}[C(d)_h + C(d)_V] \tag{23}$$

$$\mathrm{C}(0) = \frac{1}{mn}\sum_{\mathrm{i}=1}^{\mathrm{m}}\sum_{\mathrm{j}=1}^{\mathrm{n}}(\Delta h_{ij})^2 \tag{24}$$

$$\mathrm{C(d)_h} = \frac{1}{m(n-q)}\sum_{i=1}^{m}\sum_{j=1}^{n-q}\Delta h_{ij}\Delta h_{i,j+q} \tag{25}$$

$$\mathrm{C(d)_V} = \frac{1}{n(m-q)}\sum_{j=1}^{n}\sum_{i=1}^{m-q}\Delta h_{ij}\Delta h_{i+q,j} \tag{26}$$

where d is the distance between two data points, $C(d)_h$ and $C(d)_v$ represent the covariance function with x -being the horizontal direction (corresponding to m) and y -being the vertical directions (corresponding to n), respectively, m and n are the grid numbers in the two directions, and Δh is the difference between depth h and its regional mean value. According to Lemoine, et, al. (1998), we give an error of 400 m for the initial depth model JGP95E.

To evaluate the accuracy of the altimeter-derived depths, the estimated results are compared with the shipborne depths requested from our department in the region: (7° ≤ φ ≤20° ; 110 ° ≤ λ ≤120°). The data came from numerous cruises dating from 1990 to 2000. It is believed that the accuracy of the ship depth is better than 1% of the depth. Before comparison, the depths from the altimeter-derived grid are interpolated to each ship depth location using a weighted-means method. The comparisons are made at all 152,979 stations. First of all, the statistics of the ship depths themselves used for the comparison are presented in Table 1:

Table.1 Statistics of shipborne depth [Unit: m]

Year	Number	Max	Min	Mean	RMS	STD
1990	3037	4395	308	2079	2216	767
1991	2545	4448	493	2424	2571	855
1993	3833	4613	622	4173	4197	454
1994	2532	4303	258	2794	2944	929
1995	2492	4359	576	3474	3610	978
1996	2405	4441	1164	3302	3404	825
1997	13874	4463	273	2037	2272	1005
1998	9677	4276	218	2637	2722	676
1999	55314	4296	222	3775	3832	661
2000	57237	4514	463	3616	3665	596

Table 2 gives the statistics of the differences between the altimeter-derived and the shipborne depth.

Table.2 Statistics of differences between the estimated and the shipborne depths [Unit: m]

Year	Max	Min	Mean	RMS	STD	Relative error
1990	652.4	-642.3	10.6	184.1	183.8	8.9%
1991	596.1	-589.7	9.3	169.5	169.3	7.0%
1993	551.3	-553.3	12.3	132.7	132.2	3.2%
1994	550.4	-570.9	24.7	163.0	161.1	5.8%
1995	457.0	-458.9	2.9	125.7	125.6	3.6%
1996	639.6	-621.9	25.8	183.8	182.0	5.6%
1997	673.9	-684.2	-59.8	208.0	199.3	10.2%
1998	712.7	-714.1	20.0	207.1	206.2	7.9%
1999	647.0	-723.3	-12.1	158.9	158.5	4.2%
2000	465.5	-466.4	26.1	114.2	111.2	3.2%

Table.3 Statistics of discrepancies between JGP95E and the shipborne depths [Unit: m]

Year	Max	Min	Mean	RMS	STD	Relative error
1990	820.4	-825.8	35.4	224.6	221.8	10.8%
1991	890.2	-889.0	41.8	260.7	257.3	10.8%
1993	1154.7	-1163.7	114.9	272.6	247.2	6.5%
1994	1028.3	-1020.9	2.5	281.9	281.9	10.1%
1995	736.0	-748.0	16.6	246.9	246.4	7.1%
1996	915.1	-929.4	15.3	250.0	249.5	7.6%
1997	882.3	-882.6	-19.3	235.6	234.8	11.6%
1998	1026.6	-1031.6	-2.6	302.5	302.4	11.5%
1999	1392.4	-1391.2	126.1	355.4	332.2	9.4%
2000	858.9	-857.6	76.3	217.3	203.5	6.0%

To show the improvement of the new model on the others, the differences between the shipborne depths and the results from the initial depth model JGP95E and Hwang (1999) are listed in Table 3 and Table 4, respectively.

Table.4 Statistics of discrepancies between Hwang(1999) and the shipborne depths [Unit: m]

Year	Max	Min	Mean	RMS	STD	Relative error
1990	628.2	-624.8	30.9	188.4	185.8	9.1%
1991	625.1	-628.9	49.5	197.3	191.0	8.1%
1993	697.8	-752.5	25.0	204.6	203.1	4.9%
1994	731.7	-728.3	52.3	225.8	219.6	8.6%
1995	703.4	-736.2	19.2	242.7	241.9	7.0%
1996	605.9	-697.1	-33.3	220.0	217.5	6.7%
1997	654.1	-671.0	-58.6	202.9	194.2	10.0%
1998	843.9	-846.5	25.4	267.6	266.4	10.1%
1999	761.2	-764.2	-18.5	208.5	207.7	5.5%
2000	565.9	-566.1	9.9	154.9	154.6	4.3%

Making a comparison between Table 2, Table 3 and Table 4, it can be seen that the improvement of the new depth model compared to JGP95E is significant. Additionally, the computational efficiency of using Equation (17) as observation model is slightly better than that of Equation (15). It is proved from another way that the results obtained here is confident. Taking into account the existence of errors in the shipborne depths, it can be concluded that the relative error of altimetry-derived depths reaches the level of about 7%, which is better than what expected.

4 Conclusions

This article has concentrated on the recovery of depths from the altimetry-derived gravity anomalies using a modified stochastic model. The comparisons to shipborne depths have shown that the results obtained here are satisfying. It should be pointed out that such depth models from altimeter data will not be sufficiently accurate for such purposes as maritime navigation and hazard prevention, but can be very useful in many research fields of oceanography (Hwang, 1999). Future improvements will be made by combining more accurate altimetry-derived gravity anomalies and the existing sparse shipboard depths.

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